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APPLICATION OF DECOMPOSITION TECHNIQUES TO SHORT-TERM OPERATION PLANNING OF HYDROTHERMAL POWER SYSTEM

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Abstract - This paper develops a realistic model for short-term operation planning of a large scale hydrothermal power system with a high share of hydro. This problem is a large scale mixed integer program. Benders' method is employed to decompose the problem with respect to integer and continuous variables. The master problem of this method contains only integer variables and considers the unit commitment of thermal plants. The subproblem includes only continuous variables and considers the economic dispatch problem. The special structure of master and subproblems are further exploited which results in considerable reductions in the size of the problem and computation time requirement.

INTRODUCTION

The short-term operation planning of the hydrothermal power system, as modelled in this paper, is a large mixed integer programming problem. The objective of this problem, due to negligible marginal cost of hydroelectric generation, is how to use the water availability for hydroelectric generation in order to reduce the production cost of the thermal plants. The optimization horizon varies from one week to one day and the discretization intervals are chosen between one to several hours.

Literature published in this area, [1] to [10], purpose solution techniques that usually have one of the following shortcomings:

- not applicable to a real size system
- require high cpu times
- oversimplify the problem
- do not consider systems with a large proportion of hydro power

References [4],[5], an earlier part of this work, have employed Lagrangian relaxation which can consider large scale system with considerable amount of hydro and requires low cpu time. There have also been attempts to consider the hydro isolated from the total system [11],[13],[14].

In this paper, Benders' method has been employed to decompose the problem into a master problem that concerns the thermal commitment schedule and a subproblem that considers the economic dispatch problem. The commitment schedule of a large part of thermal plants can be identified without solution of master problem. Consequently, the master problem must be solved only to obtain the unit commitment schedule for a low number of thermal plants. This problem is decomposable with respect to different plants. Therefore, the master problem is further decomposed into smaller problems. This is very important since the master problem contains the integer variables.

The subproblem is further decomposed with respect to hydro and thermal systems. This makes it possible to apply fast network flow algorithm to exploit the network structure of the large number of constraints involved [11], [21]. The performance of the procedure is demonstrated on a system consisting of 30 hydro and 20 thermal plants.

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FORMULATION OF PROBLEM [4][16]

Notation

$c_f(i)$ = fixed operating cost of thermal unit i

$c_s(i)$ = start up cost of the thermal unit i

$c_v(i)$ = variable operating cost of thermal unit i

I = set of indices for the thermal plants in the system

J = set of indices for the hydro plants in the system

K = set of indices for the periods in the optimization horizon

$L(j)$ = number of segments in the power production curve of hydro plant j

$\eta_\ell(j)$ = slope of the ℓ:th segment in the power production curve of hydro plant j

$p_H(j,k)$ = power production of hydro plant j during period k

$p_D(k)$ = power demand during period k

$p_S(k)$ = spinning reserve requirement during hour k

$\underline{p}_T(i)$ = minimum power production at plant i

$\bar{p}_T(i)$ = maximum power production at plant i

$p_T(i,k)$ = power production of thermal plant i during period k

$r(j,k)$ = natural inflow to reservoir j during period k

$s(j,k)$ = spillage from reservoir j during period k

τ_{mj} = water time delay to reach reservoir j from its upstream reservoir m

$u(j,k)$ = discharge from reservoir j during period k

$u_\ell(j,k)$ = component of u(j,k) corresponding to the ℓ:th segment of power production curve for plant j

$x(j,k)$ = content of reservoir j at the begin-

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ning of period k

y(i,k) = schedule of thermal unit i during period k (1=unit in operation, 0=unit not in operation)

z(i,k) = start up schedule of thermal unit i at the beginning of period k (1=unit is started, 0=unit is not started)

A bar below a variable, such as $\underline{x}$, represents the lower bound of that variable and a bar above a variable, such as $\bar{x}$, represents the upper bound of the variable.

The objective function

The objective of the short-term operation planning of hydrothermal power system, due to the negligible marginal cost of hydroelectric generation, is the production cost of thermal plants over the operation horizon.

$$F = \sum_{k\in K} \sum_{i\in I} \{c_v(i)[p_T(i,k) - \underline{p}_T(i)] + c_p(i)\ y(i,k) + c_s(i)\ z(i,k)\} \qquad (1)$$

The production cost of a thermal plant consists of three components; start up, fixed and variable operating costs.

The cost of starting a thermal plant depends exponentially on the time for which the boiler has been cooling, but usually simplifications are made, i.e. linearization. This paper considers the model used at the Swedish dispatch centers, where the start up cost can take two distinct values; cold start value (if the boiler has been cooling for more than a certain time τ), and hot start (if the cooling time is less than τ).

Figure 1 represents typical operating cost used for the thermal plants at the Swedish dispatch centers. The operating cost has two components; a fixed component $c_f(i)$ which corresponds to unit operating cost at its minimum power production level, and a variable component which corresponds to the cost of power production between the minimum and maximum limits (slope $c_v(i)$).

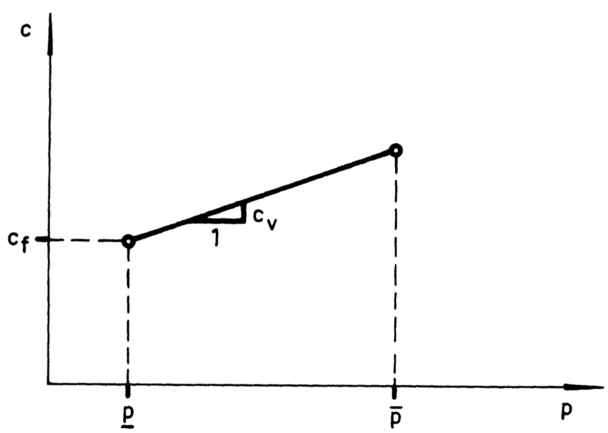

Fig. 1. Typical operating costs curve used at the Swedish dispatch centers.

Constraints

Constraints can be divided into three categories:

- o thermal constraints
- o hydro constraints
- o system constraints

Thermal constraints

These constraints reflect the physical restrictions of the thermal system:

- o The relation between the three thermal variables representing start up, operation, and power production must also be considered:

$$y(i,k) - y(i,k-1) - z(i,k) \le 0 \qquad i\in I, k\in K \quad (2)$$

$$\overline{p_T}(i)\ y(i,k) - p_T(i,k) \ge 0 \qquad i\in I, k\in K \quad (3)$$

(the lower production limit of the thermal plants considered in this paper is equal to zero).

- o Minimum up-times and down-times. For the system considered in this paper the maximum number of start ups allowed per day is equal to one (K' is the set of indices corresponding to one day)

$$\sum_{k\in K'} z(i,k) \le 1 \qquad i\in I \quad (4)$$

Hydro constraints

These constraints reflect the physical restrictions of the hydro system:

- o The dynamics of hydro reservoir can be represented by the following difference equations:

$$x(j,k+1) - x(j,k) + u(j,k) - s(j,k) - \sum_m u(m,k-\tau_{mj}) - \sum_m s(m,k-\tau_{mj}) = r(j,k) \qquad j\in J, k\in K \quad (5)$$

$$\underline{x}(j) \le x(j,k) \le \bar{x}(j) \qquad j\in j, k\in K \quad (6)$$

$$\underline{u}(j) \le u(j,k) \le \bar{u}(j) \qquad j\in j, k\in K \quad (7)$$

$\sum_m$ = sum of upstream lying power station flows

Equation (5) has a network structure. This structure has been demonstrated in Fig. 2 by considering two reservoirs over three hours. (Initial and final reservoir contents are a priori known).

- o The power production of a hydro plant is a function of water discharge as shown in Fig. 3 for a given head. This is concave and consists of L segments. η's are the slopes of different segments and depend on the reservoir head [16]. The power output for hydro station j at a given head during hour k is:

$$p_H(j,k) = \sum_{\ell=1}^{L(j)} \eta_\ell(j)\ u\ (j,k) \qquad j\in J, k\in K \quad (8)$$

where

$$u(j,k) = \sum_{\ell=1}^{L(j)} u_\ell(j,k) \qquad j \in J, k \in K \qquad (9)$$

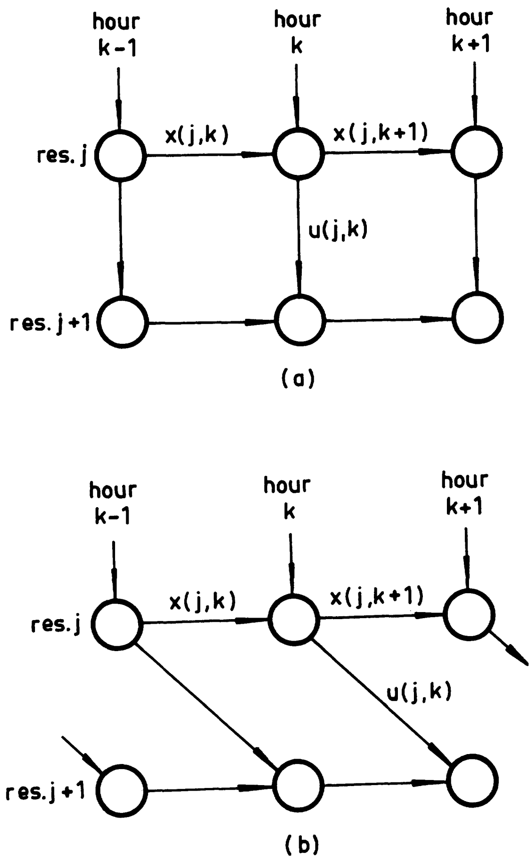

Fig. 2. The network representation of reservoir dynamics. a) no time delay; b) one hour time delay.

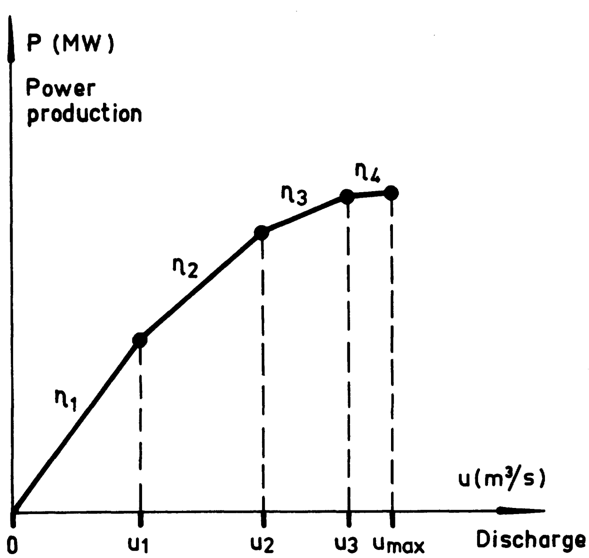

Fig. 3. Piecewise linear representation of power production curve at a given head. (number of segments is $L = 4$).

The influence of head variation in power production is rather low for short-term operation planning. It is considered by adding a couple of iterations for head correction in hydro system solution [11],[16].

System constraints

These constraints reflect the restriction due to the security requirements and transmission system:

- Spinning reserve requirement in a hydro-thermal power system, i.e. Swedish power system, is imposed on hydro

$$\sum_{j \in J} (\bar{p}_H(j) - p_H(j,k)) \geq P_s(k) \qquad k \in K \qquad (10)$$

This requirement is usually divided between different groups of hydro plants. So rather than considering one equation such as eq. (10) over the whole hydro system, a group of constraints over different parts of the hydro system is considered.

- The second requirement is to avoid transmission capacity limits. A DC model can be used to consider the transmission capacity limits in the operation planning problem:

$$p_{km} \leq \bar{p}_{km} \qquad (11)$$

where p_{km} is the power flow on a tie line and $\bar{p}_{km}$ is the upper limit for the power flow. Therefore, p_{km} is written linearly in terms of power generations at different plants [16]. For the Swedish power system, these constraints are on hydro. This is due to the fact that Sweden is a long country with the major part of its hydro production in the north and the load in the south. Therefore, the basic problem in the Swedish transmission is to transmit the northern hydro production to the south. The thermal plants are located by the load centers and they are meant for the local demand.

System demand

The system demand varies substantially over the optimization horizon. The power balance equation to meet the demand of each period will result in one constraint:

$$\sum_{i \in I} p_T(i,k) + \sum_{j \in J} p_H(j,k) = p_D(k) \quad k \in K \qquad (12)$$

BENDERS' METHOD

The short-term operation planning as formulated in eqs. (1)-(12) can be written in the following matrix form:

minimize

$$F = [c_x]^T[x] + [c_y]^T[y] \qquad (13)$$

subject to

$$[A_1][x] + [A_2][y] \geq [b] \qquad (14)$$

$$[y] \geq 0$$

$$[x] \in X \quad \text{(integer)} \tag{15}$$

where vector [x] corresponds to integer variables and vector [y] corresponds to continuous variables. $[c_x]$ and $[c_y]$ are the cost coefficient vectors in the objective corresponding to integer and continuous variables respectively. Matrices $[A_1]$ and $[A_2]$ are coefficient matrices and vector [b] contains the right hand sides of the constraints.

This problem can be considered as a two stage decision process [17]. In the first stage, a feasible decision [x*] is assumed for vector [x]. In the second stage, a decision [y*] is calculated as the optimal solution to the following problem:

minimize

$$[c_y]^T[y] \tag{16}$$

subject to

$$[A_2][y] \geq [b] - [A_1][x^*] \tag{17}$$

$$[y] \geq 0 \tag{18}$$

The second stage problem is a function of decision [x] taken in the first stage. Therefore, the global optimum can be written as:

minimize

$$[c_x]^T[x] + \sigma[x] \tag{19}$$

subject to

$$[x] \in X \tag{20}$$

The function $\sigma([x])$ reflects information about the future consequence of decision [x], and the second stage is consequently mapped into the first one. The Benders' method is a scheme for building up $\sigma([x])$ with a required accuracy by iterating between the first and second stage problems. In each iteration, a constraint (Benders' cut) is added to the first stage problem eqs. (19)-(20), using the dual values of the second stage problem eqs. (16)-(18), say $[\pi]$. Therefore, the first stage problem is converted to [16],[17],[18]:

minimize

$$[c_x]^T[x] + \sigma \tag{21}$$

subject to

$$[\pi]_n^T([b] - [A_1][x]) \leq \sigma \quad n=1,\ldots,L \tag{22}$$

$$[x] \in X \tag{23}$$

L is the iteration number.

Fig. 4 represents application of Benders' method. The master problem, eqs. (21)-(23), is an integer program and it is solving for the unit commitment schedule of thermal plants, and feeding them to the subproblem, eqs. (16)-(18). The subproblem is a continuous program with fixed thermal commitment schedules. The dual values of the subproblem are fed back to the master problem and introduce a new constraint (Benders' cut) to this problem. This process is continued until the upper and lower bounds, available in this method, converge [16],[17],[18].

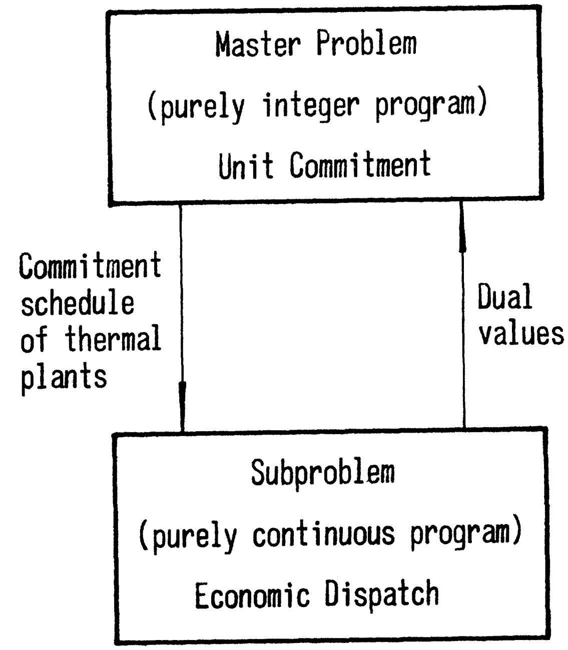

Fig. 4. Diagram representing application of Benders' method to short-term operation planning.

Benders' cut

The Benders' cut is generated from the dual values of the subproblem. Actually, the only dual values required to generate this cut are the ones corresponding to the coupling constraints between continuous and integer variables, eq. (3).

The dual of the second stage problem, eqs. (16)-(18), must be solved to evaluate the dual values. Since the coefficient matrix is transposed in the dual problem, the special characteristics of this matrix, corresponding to the large number of hydro constraints with network structure, does not hold anymore.

To avoid such a problem that destroys the main purpose of the decomposition technique, the subproblem (16)-(18) is solved directly, and the dual quantities, corresponding to the coupling constraints, are evaluated from the general definition of dual variables, i.e.:

$$\text{dual variable } i = \frac{d(\text{objective function})}{d(\text{right hand side})} = \frac{dF}{db_i} \tag{24}$$

Let us consider increasing the right hand side of the constraint for plant i during hour k in eq. (3). Since the first term of this equation is fixed (because the commitment schedules of thermal units are fixed), the power production of plant i during hour k is decreased by one unit. In order to meet the load, the power production of the last plant during hour k has to be increased by one unit. Consequently, the objective function is decreased by the variable operation cost of unit i and increased by the marginal prices of the hour k, i.e.:

$$\pi(i,k) = \lambda(k) - c_v(i) \qquad (25)$$

where

$\pi(i,k)$ = the dual variable corresponding to the constraint for unit i during hour k, eq. (4)
$\lambda(k)$ = marginal price of hour k
$c_v(i)$ = variable operation cost of unit i

It is very easy to verify the dual values because the quantities on the right hand side of eq. (25) are both known.

The subproblem

The subproblem in Benders' method actually considers the problem of economic dispatch in hydrothermal system when the thermal unit commitment schedules are fixed. It is a staircase problem as shown in Fig. 5. The block, corresponding to the hydro constraints, with network structure, constitutes the dominating part of this problem.

Dantzig-Wolfe decomposition [12],[19] or Lagrangian relaxation [5],[20] techniques can be applied, which makes it possible to exploit the network structure of the problem using network flow algorithms [21]. These algorithms are more than 100 times faster than standard methods [21].

Practical data available (e.g. initial marginal prices which can normally be approximated or built up heuristically) are used to speed up the solution techniques.

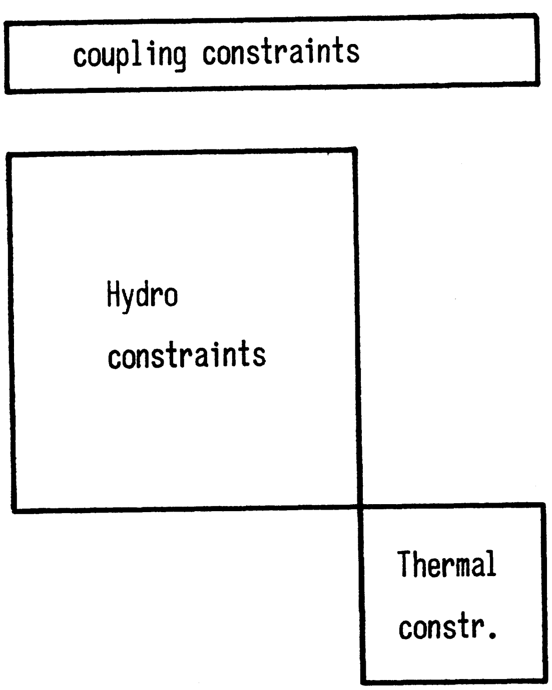

Fig. 5. The staircase structure of subproblem.

The master problem

A more detailed formulation of the master problem, eqs. (21)-(23), is as follows:

minimize

$$\sigma + \sum_{k\in K}\sum_{i\in I} [c_f(i)\ y(i,k) + c_s(i)\ z(i,k)] \qquad (26)$$

subject to

$$\sum_{k\in K'} z(i,k) \le 1 \qquad i\in I \qquad (27)$$

$$y(i,k) - y(i,k-1) - z(i,k) \le 0 \qquad i\in I, k\in K \qquad (28)$$

$$[-\bar{p}_T(i)\ y(i,k)]\Big|_{i\in I,\ k\in K} [\pi]^n \le \sigma \ \text{(Benders' cuts)} \quad n=1,\dots,t \qquad (29)$$

It was indicated in the previous subsection that the dual variables were the surpluses of the marginal prices from the variable operation costs of different thermal plants, eq.(25). Benders' cuts, eq. (29), reflect these marginal prices to the objective, eq. (26), to cover for the start up and fixed operation costs of different thermal plants. Therefore, this is decomposable with respect to different plants as follows:

$$\sigma_i + \sum_{k\in K} [c_f(i)\ y(i,k) + c_s(i)\ z(i,k)] \qquad (30)$$

subject to

$$\sum_{k\in K'} z(i,k) \le 1 \qquad (31)$$

$$y(i,k) - y(i,k-1) - z(i,k) \le 0 \qquad k\in K \qquad (32)$$

$$[-\bar{p}_T(i)\ y(i,k)]\Big|_{k\in K} [\pi]^n \le \sigma_i \ \text{(Benders' cuts)} \quad n=1,\dots,t \qquad (33)$$

This reduces the master problem which contains all the integer variables to small integer programs, each corresponding to a thermal plant.

In the master problem actually the marginal prices of different hours are compared with start up, fixed and variable operating costs of different plants. Therefore, any plant that has coverage with respect to marginal prices is scheduled. If lower and upper bounds for the marginal prices are defined during optimization horizon [5],[16], then all the plants that have cost coverage with the minimum of the marginal prices are considered scheduled and the ones which do not have cost coverage even with maximum of the marginal prices are considered not scheduled. Therefore, only the plants that fall between the maximum and minimum must be considered in the master program. This reduces the size of the master problem considerably. Discrete dynamic programming technique is employed for the solution of the master problem.

CONCLUSIONS AND TEST RESULTS

The Swedish Power System has been used to demonstrate the practical application of the purposed method. The largest test system employed [16] consisted of 30 hydro plants on five rivers and 20 thermal plants (nuclear, fossil fuelled, cogeneration, and gas turbine). Convergence was normally obtained within fifteen iterations of Benders' method with a tolerance of 2%. The total computation time required for a daily scheduling of the above system was about 4 minutes on Vax 11-780. Figure 6 demonstrates a typical example of hydro production, thermal production and marginal prices obtained with respect to system demand.

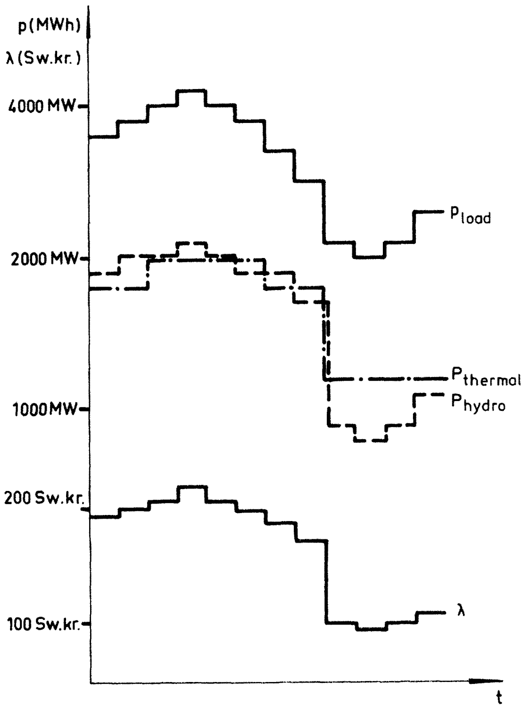

Fig. 6. Typical marginal prices, hydro, and thermal productions obtained with respect to system demand.

The Benders' method has a slightly higher efficiency than the Lagrangian relaxation technique considered in an earlier part of this work [5],[16]. Both of these methods are capable of producing quite better solutions than heuristic methods [5]. Heuristic methods require lower computation times but they do not guarantee a near optimum solution. These methods normally produce solutions that are more than 5% away from optimum. The Benders' or Lagrangian relaxation techniques can always produce solutions that are within 2% from optimum.

This research was coordinated with the project in the Swedish utilities on software development of operations planning. The hydro subproblem using predefined marginal prices has been implemented for a purely hydro power utility and has proved to be very useful for the dispatch engineers.

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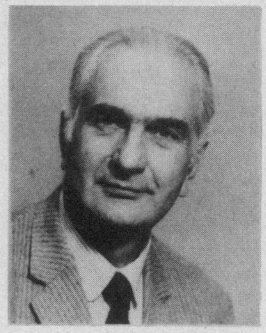

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